

The solution to Thermology(Chapter2, 4, 6)

Shuo Fang, PB17071424

email: fangshuo@mail.ustc.edu.cn

Website: home.ustc.edu.cn/~fangshuo

2020 年 5 月 25 日

Content

1 Ch2	1
2 Ch4	14
3 Ch6	19
4 Reference	21

Abstract

This note is the solution to some chapters(Chapter 2, 4, 6) of 《Thermology》 (《热学》, Yumin Zhang).

1 Ch2

2.1 (10 points)

a (3 points, final result 1 point) For isobaric process, one have

$$W = \int_{V_i}^{V_f} p dV = p\Delta V = pV_f - pV_i = 10^5 Pa \times (2.0 - 1.0)m^3 = 1.0 \times 10^5 J \quad (1)$$

b (1 point)For ideal gas, one have

$$C = \frac{pV}{T} = \frac{p_i V_i}{T_i} \equiv const \quad (2)$$

(4 points, final result 1 point)During the isothermal process $T = T_i$, thus,

$$W = \int_{V_i}^{V_f} p dV = \int_{V_i}^{V_f} \frac{CT}{V} dV = p_i V_i \ln\left(\frac{V_f}{V_i}\right) = 10^5 Pa \times 1m^3 \times \ln 2 = 6.93 \times 10^4 J \quad (3)$$

c (2 points)For isochoric process, the volume will remain the same: $dV = 0$, thus the work is zero.

2.2 (10 points) For isothermal process, we have:

$$\frac{pV}{T} = \frac{pV}{T_i} = \frac{p_i V_i}{T_i} = nR \equiv \text{const} \quad (4)$$

由

$$\begin{aligned} W &= - \int_{V_i}^{V_f} p dV = - \int_{V_i}^{V_f} \frac{p_i V_i}{V} dV = -p_i V_i \ln \frac{V_f}{V_i} = -T_i nR \ln \frac{V_f}{V_i} \\ \Rightarrow V_f &= V_i \exp\left(-\frac{W}{T_i nR}\right) = 5.0L \times \exp\left(-\frac{4500}{293 \times 0.25 \times 8.314}\right) = 3.09 \times 10^{-3}L \simeq 3.1 \times 10^{-3}L \end{aligned} \quad (5)$$

Note: The work the system does to the environment is: $dW = p dV$; while the work the environment does to the system is: $dW' = -p dV$.

2.3 (10 points) For the mentioned gas during the isothermal process, we have:

$$W = - \int_{V_i}^{V_f} dV p = - \int_{V_i}^{V_f} dV \frac{RT}{V-b} = -RT \ln \frac{V_f - b}{V_i - b} \quad (6)$$

While for the ideal gas, we have:

$$W' = - \int_{V_i}^{V_f} dV p = - \int_{V_i}^{V_f} dV \frac{RT}{V} = -RT \ln \frac{V_f}{V_i} \quad (7)$$

As the volume is getting larger, $V_f > V_i$, thus one gets $\frac{V_f - b}{V_i - b} > \frac{V_f}{V_i} > 1$ and

$$W < W' \quad (8)$$

means the work is less.

2.4 (10 points) For 1 mol van der Waals gas, one has

$$\left(p + \frac{a}{v^2}\right)(v - b) = RT \quad (9)$$

so,

$$W = - \int_i^f dV p = - \int_{V_i}^{V_f} dV \left(\frac{RT}{V-b} - \frac{a}{V^2}\right) = RT \ln \frac{V_i - b}{V_f - b} + a\left(\frac{1}{V_i} - \frac{1}{V_f}\right) \quad (10)$$

2.5 (10 points) During the ab process, the p-V relation is given by equation:

$$p = -\frac{50}{3}V + \frac{310}{3} \quad (11)$$

thus,

$$\begin{aligned} W_{ab} &= - \int_{V_a}^{V_b} p dV = - \int_{V_a}^{V_b} dV \left(-\frac{50}{3}V + \frac{310}{3}\right) \times 100J = 100 \times \left(\frac{25}{3} \times 21 - \frac{310}{3} \times 3\right) \times 10^{-6}J \\ &= -1.35 \times 10^{-2}J \end{aligned} \quad (12)$$

During the bc process:

$$W_{bc} = - \int_{V_b}^{V_c} p dV = p_b(V_b - V_c) = 20 \times 10^2 Pa \times (5 - 2) \times 10^{-6} m^3 = 6 \times 10^{-3} J \quad (13)$$

During the ca process, given the volume does not change, $dV = 0$.

$$W_{ca} = - \int_{V_c}^{V_a} p dV = 0 J \quad (14)$$

And the total work environment do to system in this circuit is:

$$W = W_{ab} + W_{bc} + W_{ca} = -7.5 \times 10^{-3} J \quad (15)$$

2.6 (10 points) For the adiabatic process, the state equation is

$$pV^\gamma = Const = p_i V_i^\gamma = p_f V_f^\gamma \quad (16)$$

Then the work is:

$$\begin{aligned} W &= - \int_{V_i}^{V_f} p dV = - \int_{V_i}^{V_f} dV \frac{p_i V_i^\gamma}{V^\gamma} = - p_i V_i^\gamma \frac{V^{-\gamma+1}}{-\gamma+1} \Big|_{V_i}^{V_f} \\ &= \frac{p_i V_i^\gamma V_f^{-\gamma+1} - p_i V_i}{\gamma-1} = \frac{p_f V_f^\gamma V_f^{-\gamma+1} - p_i V_i}{\gamma-1} = \frac{p_f V_f - p_i V_i}{\gamma-1} \end{aligned} \quad (17)$$

2.7 (10 points) For isothermal process, $dT = 0$

$$\begin{aligned} W &= - \int p dV = - \int p \left(\frac{\partial V}{\partial T} \right)_p dT + \left(\frac{\partial V}{\partial p} \right)_T dp = - \int p \left(\frac{\partial V}{\partial p} \right)_T dp \\ &= \int p V \beta dp = \int p \frac{m}{\rho} \beta dp = \frac{m\beta}{2\rho} p^2 \Big|_0^{10^7} \\ &= \frac{0.1 \times 6.75 \times 10^{-12} \times 10^{14}}{2 \times 10^4} J = 3.375 \times 10^{-3} J \end{aligned} \quad (18)$$

Note: $\beta \equiv -\frac{1}{V} \left(\frac{\partial V}{\partial p} \right)_T$ is the constant temperature compression coefficient.

2.8 (10 points) (a) If the volume remains constant, we will use c_V :

$$Q_1 = C_V \Delta T = \frac{c_p}{\gamma} m \Delta T = \frac{1}{1.41} \times 9.963 \times 10^2 \times 27 \times 1.29 \times 20 J = 4.92 \times 10^5 J \quad (19)$$

(b)

$$Q_2 = C_p \Delta T = c_p m \Delta T = \frac{Q_1}{\gamma} = 4.92 \times 10^5 \times 1.41 J = 6.94 \times 10^5 J \quad (20)$$

(c) Since the room is connected with the environment, the pressure and volume remain constant, $pV = \frac{m}{\mu} RT \equiv const$. Then one has $mT = const$. And this process will be pressure constant with the quantity of gas changes.

$$\begin{aligned} Q_3 &= \int m c_p dT = \int \frac{m_0 T_0}{T} c_p dT = m_0 T_0 c_p \ln \frac{T_1}{T_0} \\ &= 1.29 \times 27 \times 996.3 \times 273 \times \ln \frac{293}{273} J = 6.70 \times 10^5 J \end{aligned} \quad (21)$$

Note: The origin comes from our textbook page 43, example 2.2.

2.9 (10 points)

$$Q = \int ncdT = \int nA\left(\frac{T}{T_D}\right)^3 dT = \frac{nA(T_f^4 - T_i^4)}{4T_D^3} = \frac{500 \times 1.94 \times 10^3 \times (625 - 1)}{4 \times 300^3} J = 5.60J \quad (22)$$

2.10 (10 points) The heat released equals to that absorbed.

$$\begin{aligned} c_{copper}m_{copper}(T_{1i} - T_f) &= c_{water}m_{water}(T_f - T_{2i}) \\ \Rightarrow c_{copper} &= \frac{c_{water}m_{water}(T_f - T_{2i})}{m_{copper}(T_{1i} - T_f)} = \frac{4.184 \times 10^3 \times 5 \times (22.3 - 20)}{1 \times (100 - 22.3)} J/(kg \cdot K) \\ &= 6.19 \times 10^2 J/(kg \cdot K) \end{aligned} \quad (23)$$

2.11 (10 points) For this process, take the gas in the box afterwards as a system, and one can get the difference of the internal energy of the gas:

$$U - U_0 = W + Q \quad (24)$$

And it is an adiabatic process, $Q = 0$; given the box is small, the pressure can be taken to be constant; the work atmosphere done to gas is:

$$W = p\Delta V = p_0V_0 \quad (25)$$

but the gas did no work to the "substance" in the box. As a result, the change of internal energy is only determined by (25) and one gets the result:

$$U - U_0 = W + Q = p_0V_0 \quad (26)$$

2.12 (10 points) According to 1st law of thermodynamics, one has

$$U_f - U_i = Q_{if} + W_{if} = 50J - 20J = 30J \quad (27)$$

(a)

$$\begin{aligned} \Delta U = W + Q' &= -5J + Q' = 30J \\ Q' &= 35J \end{aligned} \quad (28)$$

(b)

$$\begin{aligned} -\Delta U = W + Q &= 10J + Q = -30J \\ Q &= -40J \end{aligned} \quad (29)$$

(c) There is a paradox between the main stem and sub-stem.

Note: In $\Delta U = Q + W$, Q is the heat absorbed from the environment and W is the work environment does to the system.

2.13 (10 points)

$$C_V = \left(\frac{\partial U}{\partial T}\right)_V = C - Bp - BT\left(\frac{\partial p}{\partial T}\right)_V + Ap\left(\frac{\partial p}{\partial T}\right)_V$$

And, for $dV = 0$:

$$\begin{aligned} V &= V_0 - Ap + BT \\ \Rightarrow dV &= -Adp + BdT \\ \Rightarrow \left(\frac{\partial p}{\partial T}\right)_V &= \frac{B}{A} \end{aligned} \quad (30)$$

Then,

$$\begin{aligned} C_V &= C - Bp - T\frac{B^2}{A} + Bp \\ &= C - T\frac{B^2}{A} \end{aligned} \quad (31)$$

As for C_p , one can get:

$$\begin{aligned} H &= U + pV = CT - BpT + \frac{1}{2}Ap^2 + pV_0 - Ap^2 + BpT \\ &= CT + pV_0 - \frac{1}{2}Ap^2 \end{aligned} \quad (32)$$

$$\begin{aligned} C_p &= \left(\frac{\partial H}{\partial T}\right)_p \\ &= C \end{aligned} \quad (33)$$

Thus, the difference between the two heat capacity:

$$C_p - C_V = T\frac{B^2}{A} \quad (34)$$

Note: one can get the differential form of enthalpy using Legendre transformation:

$$\begin{aligned} \Rightarrow dH &= dU + pdV + Vdp \\ &= dQ + Vdp \\ &= \left(\frac{\partial H}{\partial T}\right)_p dT + Vdp \\ &= C_p dT + Vdp \end{aligned} \quad (35)$$

2.14 (10 points)

2.15 (10 points) It is a (p, V) system, where all of the parameters of thermodynamic state can be expressed by functions p, V

(a)

$$\begin{aligned} dU &= dQ + dW \\ &= dQ - pdV \end{aligned}$$

$$\begin{aligned} \Rightarrow dQ &= pdV + dU \\ &= pdV + \left(\frac{\partial U}{\partial V}\right)_p dV + \left(\frac{\partial U}{\partial p}\right)_V dp \\ &= \left(\frac{\partial U}{\partial p}\right)_V dp + \left(p + \left(\frac{\partial U}{\partial V}\right)_p\right) dV \end{aligned} \quad (36)$$

(b) One has

$$\begin{aligned} \alpha &= \frac{1}{V} \left(\frac{\partial V}{\partial T}\right)_p \\ \beta &= -\frac{1}{V} \left(\frac{\partial V}{\partial p}\right)_T \\ \left(\frac{\partial V}{\partial T}\right)_p \left(\frac{\partial T}{\partial p}\right)_V \left(\frac{\partial p}{\partial V}\right)_T &= -1 \end{aligned} \quad (37)$$

Then, take $U = U(T, V)$

$$\begin{aligned} \left(\frac{\partial U}{\partial p}\right)_V &= \left(\frac{\partial U}{\partial T}\right)_V \left(\frac{\partial T}{\partial p}\right)_V = C_V \left(\frac{\partial T}{\partial p}\right)_V = -C_V \frac{1}{\left(\frac{\partial V}{\partial T}\right)_p \left(\frac{\partial p}{\partial V}\right)_T} \\ &= -C_V \frac{\left(\frac{\partial V}{\partial p}\right)_T}{\left(\frac{\partial V}{\partial T}\right)_p} \\ &= C_V \frac{\beta}{\alpha} \end{aligned} \quad (38)$$

(c) Let us set $dp = 0$ for an isobaric process and choose (p, T) as parameters of thermodynamic state:

$$\begin{aligned} dQ &= \left(p + \left(\frac{\partial U}{\partial V}\right)_p\right) dV \\ \Rightarrow C_p dT &= \left(p + \left(\frac{\partial U}{\partial V}\right)_p\right) \left(\frac{\partial V}{\partial p}\right)_T dp + \left(\frac{\partial V}{\partial T}\right)_p dT \\ &= \left(p + \left(\frac{\partial U}{\partial V}\right)_p\right) \left(\frac{\partial V}{\partial T}\right)_p dT \\ \Rightarrow \left(\frac{\partial U}{\partial V}\right)_p &= \frac{C_p}{\left(\frac{\partial V}{\partial T}\right)_p} - p \\ &= \frac{C_p}{V\alpha} - p \end{aligned} \quad (39)$$

2.16 (10 points)

$$C_{V,m} = \left(\frac{\partial U}{\partial T}\right)_V = c \quad (40)$$

And the enthalpy is

$$H = H(p, T) = u + pV = cT - \frac{a}{v} + pV \quad (41)$$

Thus,

$$C_{p,m} = \left(\frac{\partial H}{\partial T}\right)_p = c + \frac{a}{v^2} \left(\frac{\partial v}{\partial T}\right)_p + p \left(\frac{\partial v}{\partial T}\right)_p \quad (42)$$

And use the equation of gas state:

$$\begin{aligned} \left(p + \frac{a}{v^2}\right)(v - b) &= RT \\ \Rightarrow \left(\frac{\partial T}{\partial v}\right)_p &= \frac{1}{R} \left(p - \frac{a}{v^2} + \frac{2ab}{v^3}\right) \\ \Rightarrow \left(\frac{\partial v}{\partial T}\right)_p &= \frac{R}{p - \frac{a}{v^2} + \frac{2ab}{v^3}} \end{aligned} \quad (43)$$

And one can get $C_{p,m}$:

$$\begin{aligned} C_{p,m} &= c + \frac{RT}{v - b} \frac{R}{p - \frac{a}{v^2} + \frac{2ab}{v^3}} \\ &= c + \frac{R^2 T}{p(v - b) - \left(\frac{a}{v} - \frac{2ab}{v^2}\right) + \frac{2ab}{v^2} - \frac{2ab^2}{v^3}} \\ &= c + \frac{R^2 T}{\left(p + \frac{a}{v^2}\right)(v - b) - \frac{2ab^2 - 4abv + 2av^2}{v^3}} \\ &= c + \frac{R^2 T}{RT - 2a \frac{(b-v)^2}{v^3}} \end{aligned} \quad (44)$$

2.17 (10 points)

$$\begin{aligned} \Delta U &= Q_{s \rightarrow l} + Q_{0 \rightarrow 100} + Q_{l \rightarrow g} - p\Delta V \\ &= L_1 m + Cm\Delta t + L_2 m - p\Delta V \\ &= (3.338 \times 10^5 \times 18 \times 10^{-3} + 4.184 \times 10^3 \times 18 \times 10^{-3} \times 100 \\ &\quad + 2.257 \times 10^6 \times 18 \times 10^{-3} - 1.01325 \times 10^5 \times 0.018 \times (1.673 - 1.043 \times 10^{-3}))J \\ &= 5.112 \times 10^4 J \end{aligned} \quad (45)$$

As for the change of enthalpy, the pressure remains the same:

$$\begin{aligned} \Delta H &= Q_{s \rightarrow l} + Q_{0 \rightarrow 100} + Q_{l \rightarrow g} \\ &= 5.416 \times 10^4 J \end{aligned} \quad (46)$$

2.18 (10 points) (a)

$$W = \int pdV = \int \frac{RT}{V} dV = RT \ln 2 = RT_i \ln 2 \quad (47)$$

since $dU = C_{V,m} dT = 0$, the change of the heat equals to the work, thus the heat absorbed equals to $RT_i \ln 2$.

(b)

$$W = \int pdV = pV_i = RT_i \quad (48)$$

And the change of the internal energy is:

$$\begin{aligned} Q - W = \Delta U &= \int C_V dT = \frac{3}{2}R\Delta T = \frac{3R}{2}(2T_i - T_i) = \frac{3RT_i}{2} \\ \Rightarrow Q &= \frac{5R_i}{2} \end{aligned} \quad (49)$$

2.19 (10 points) (1) During the isobaric process,

$$W_1 = \int p dV = p\Delta V = p(V_f - V_i) \quad (50)$$

$$\begin{aligned} Q_1 = W_1 + C_V\Delta T &= p(V_f - V_i) + \frac{5R}{2}p \frac{V_f - V_i}{R} \\ &= \frac{7}{2}p(V_f - V_i) \end{aligned} \quad (51)$$

$$\frac{W_1}{Q_1} = \frac{2}{7} \quad (52)$$

(2) During the isothermal process, $dU = 0$, thus

$$\frac{W_2}{Q_2} = 1 \quad (53)$$

2.20 (10 points) (a) During the isobaric process,

$$\begin{aligned} Q &= W + C_V m \Delta T = p\Delta V + C_V m \Delta T = (\nu R + C_V m) \Delta T \\ &= \left(\frac{1000}{28} \times 2 + 5 \times \frac{1000}{28} \right) \times 120 \text{ cal} = 3.00 \times 10^4 \text{ cal} = 1.255 \times 10^5 \text{ J} \end{aligned} \quad (54)$$

(b)

$$\Delta U = C_V m \Delta T = \frac{1000}{28} \times 5 \times 120 \times 4.184 \text{ J} = 8.966 \times 10^4 \text{ J} \quad (55)$$

(c)

$$W = p\Delta V = \nu \Delta T = \frac{1000}{28} \times 2 \times 120 \times 4.184 \text{ J} = 3.586 \times 10^4 \text{ J} \quad (56)$$

(d) If it was isochoric process, one has $dU = C_V dT = \delta Q$ and then

$$Q = \Delta U = C_V m \Delta T = 8.966 \times 10^4 \text{ J} \quad (57)$$

2.21 (10 points) During this process,

$$\begin{aligned} C_V dT &= dU = C dT - p dV \\ \Rightarrow C &= p \frac{\partial V}{\partial T} + C_V \end{aligned} \quad (58)$$

And according to:

$$\begin{aligned} pV^2 &= a^2 \\ pV &= \nu RT \\ \Rightarrow \frac{\partial V}{\partial T} &= \left(\frac{\partial V}{\partial p} \right)_T \left(\frac{\partial p}{\partial T} \right)_V = -\frac{a}{2p^{\frac{3}{2}}} \cdot \frac{2\nu^2 R^2 T}{a^2} \\ &= -\frac{\nu R V}{ap^{\frac{1}{2}}} = -\frac{\nu R}{p} \end{aligned} \quad (60)$$

Thus,

$$C = C_V - \nu R \quad (61)$$

As a result,

$$c_{n,m} = \frac{C_{n,V} - R}{\mu} \quad (62)$$

The meaning of the $c_{n,m}$ is the heat capacity per mass of unit amount of substance.

2.22 (10 points) For 1mol van der waals gas, it has the equation of state:

$$(p + \frac{a}{v^2})(v - b) = RT \quad (63)$$

Then for the adiabatic process, $dQ = 0$ so that $dU = dQ - pdV = -pdV$.

As:

$$\begin{aligned} dQ - pdV &= dU = TdS - pdV \\ &= T((\frac{\partial S}{\partial V})_T dV + (\frac{\partial S}{\partial T})_V dT) - pdV \\ &= [T(\frac{\partial S}{\partial V})_T - p]dV + T(\frac{\partial S}{\partial T})_V dT \end{aligned} \quad (64)$$

and because $dF = -SdT - pdV$, given the dF is a total derivative, there is no curl.

$$-(\frac{\partial S}{\partial V})_T = -(\frac{\partial p}{\partial T})_V \quad (65)$$

This is one of the so-called Maxwell relations.

And one has

$$C_V = T(\frac{\partial S}{\partial T})_V \quad (66)$$

Then one can derive the expression of dU during adiabatic process:

$$\begin{aligned} -pdV &= dU = [T(\frac{\partial p}{\partial T})_V - p]dV + C_V dT = [T(\frac{\partial p}{\partial T})_V - p]dV + C_V dT \\ &\Rightarrow C_V dT + [T\frac{R}{v-b} - p]dv = -pdv \\ &\Rightarrow C_V dT + T\frac{R}{v-b} dv = 0 \\ &\Rightarrow \ln T + \frac{R}{C_V} \ln(v-b) = C_1 \\ &\Rightarrow T(v-b)^{\frac{R}{C_V}} = C \end{aligned} \quad (67)$$

2.23 (10 points)

$$\begin{aligned} dU &= C_V dT = -pdV \\ &\Rightarrow -pdV = C_V \frac{1}{R} ((V-b)dp + pdV) \\ &\Rightarrow (1 + \frac{C_V}{R})pdV + \frac{C_V}{R}(V-b)dp = 0 \\ &\Rightarrow \ln(p^{\frac{C_V}{R}}(V-b)^{1+\frac{C_V}{R}}) = C_1 \\ &\Rightarrow p(V-b)^{1+\frac{R}{C_V}} = C \\ &\text{or } T(V-b)^{\frac{R}{C_V}} = C \end{aligned} \quad (68)$$

2.24 (10 points) For the ideal gas:

$$pV = \nu RT \Rightarrow dT = \frac{1}{\nu R}(pdV + Vdp) \quad (69)$$

and

$$C_p - C_V = \nu R \quad (70)$$

For this specific process:

$$\begin{aligned} dU &= C_V dT = dQ - pdV = C dT - pdV \\ &\Rightarrow \frac{C_V - C}{C_p - C_V}(pdV + Vdp) = -pdV \\ &\Rightarrow \left(1 + \frac{C_V - C}{C_p - C_V}\right)pdV + \frac{C_V - C}{C_p - C_V}Vdp = 0 \\ &\Rightarrow (C_p - C) \ln V + (C_V - C) \ln p = C_1 \\ &\Rightarrow pV^{\frac{C_p - C}{C_V - C}} = pV^n = \text{const} \end{aligned} \quad (71)$$

where $n = \frac{C_p - C}{C_V - C}$.

2.25 (10 points) (a) For the right gas, it is an adiabatic process, thus: $pV^\gamma = p_0V_0^\gamma$ and for one-atom gas, $\gamma = \frac{5}{3}$.

$$\begin{aligned} W &= - \int pdV = - \int \frac{p_0V_0^\gamma}{V^\gamma} dV \\ &= - \frac{p_0V_0^\gamma(V_f^{1-\gamma} - V_0^{1-\gamma})}{1-\gamma} = \frac{p_fV_f - p_0V_0}{\gamma - 1} \\ &= \frac{p_f\left(\frac{p_0}{p_f}\right)^{\frac{1}{\gamma}}V_0 - p_0V_0}{\gamma - 1} = \frac{1.013 \times 10^5 \times (7.59 \times \left(\frac{1}{7.59}\right)^{\frac{3}{5}} - 1) \times 54 \times 10^{-3} \text{ J}}{\frac{5}{3} - 1} \\ &= 1.025 \times 10^4 \text{ J} \end{aligned} \quad (72)$$

(b)

$$\begin{aligned} T_{r,f} &= \frac{p_{r,f}V_{r,f}}{p_{r,0}V_{r,0}}T_{r,0} = \frac{p_{r,f}\left(\frac{p_{r,0}}{p_{r,f}}\right)^{\frac{1}{\gamma}}V_{r,0}}{p_{r,0}V_{r,0}}T_{r,0} = \left(\frac{p_{r,f}}{p_{r,0}}\right)^{1-\frac{1}{\gamma}}T_{r,0} \\ &= 7.59^{1-\frac{3}{5}} \times 273\text{K} = 614\text{K} \end{aligned} \quad (73)$$

(c) The left pressure equals to the right and the total volume remains unchanged.

$$\begin{aligned} T_{l,f} &= \frac{p_{l,f}V_{l,f}}{p_{l,0}V_{l,0}}T_{l,0} = \frac{p_{r,f}(2V_{r,0} - V_{r,f})}{p_{r,0}V_{r,0}}T_{r,0} = 2\frac{p_{r,f}}{p_{r,0}}T_{r,0} - T_{r,f} \\ &= 2 \times 7.59 \times 273\text{K} - 614\text{K} = 3530\text{K} \end{aligned} \quad (74)$$

(d) Since the total gas remains the same volume and the heat absorbed by the left gas equals to total heat absorbed, which also equals to the change of total internal energy.

$$\begin{aligned} Q &= \Delta U = \Delta U_l + \Delta U_r = \nu(C_{V,m}\Delta T_l + C_{V,m}\Delta T_r) \\ &= \frac{54}{22.4} \times \frac{3}{2} \times 8.314 \times (3530 + 614 - 2 \times 273)\text{J} = 1.08 \times 10^5 \text{ J} \end{aligned} \quad (75)$$

where $C_{V,m} = \frac{3}{2}R$.

2.26 (10 points) (a) Since the exponent of adiabatic process γ is 1.40, one gets:

$$T = 2\pi\sqrt{\frac{mV}{\gamma pA^2}} = 2\pi\sqrt{\frac{0.01 \times 5 \times 10^{-3}}{1.40 \times 1.013 \times 10^5 \times 10^{-8}}} s = 1.180s \quad (76)$$

(b) This is a harmonic oscillator. And the k is

$$k = \gamma A^2 \frac{p}{V} \quad (77)$$

Then, the work the elastic force does is:

$$W = -\int_0^L f dx = -\int_0^L kx dx = -\frac{1}{2}kL^2 \quad (78)$$

And the work the gravity does is:

$$W' = mgL \quad (79)$$

And the total work equals to 0, thus,

$$L = \frac{2mg}{k} = \frac{2mgV}{\gamma A^2 p} = \frac{2 \times 10^{-2} \times 9.8 \times 5 \times 10^{-3}}{1.40 \times 10^{-8} \times 1.013 \times 10^5} m = 0.691m \quad (80)$$

2.27 (10 points)

$$\gamma = \left(\frac{2\pi}{T}\right)^2 \frac{mV}{pA^2} = \left(\frac{2\pi}{0.834}\right)^2 \frac{16.65 \times 10^{-3} \times 5270 \times 10^{-6}}{9.6525 \times 10^4 \times (2.01 \times 10^{-4})^2} = 1.277 \quad (81)$$

2.28 (10 points) For the Van der Waals gas, one can get the equation of the inverse curve from our textbook:

$$T = \frac{2a(v-b)^2}{Rbv^2} \quad (82)$$

And use the equation of state for 1 mol gas:

$$\left(p + \frac{a}{v^2}\right)(v-b) = RT \quad (83)$$

One get:

$$\begin{aligned} p &= \frac{2a(v-b)}{bv^2} - \frac{a}{v^2} = \frac{2a}{b}\left(\frac{1}{v} - \frac{b}{v^2}\right) - \frac{a}{v^2} \\ \Rightarrow \left(\frac{\partial p}{\partial v}\right)_T &= \frac{2a}{b}\left(-\frac{1}{v^2} + \frac{2b}{v^3}\right) + \frac{2a}{v^3} \\ \Rightarrow v_c &= 3b \\ \Rightarrow p_{max} &= \frac{4a}{9b^2} - \frac{a}{9b^2} = \frac{a}{3b^2} \end{aligned} \quad (84)$$

2.29 (10 points) The maximum inverse temperature can be given by appointing $p = 0$:

$$\begin{aligned} p &= -21 + 5.44T - 0.132T^2 = 0 \\ \Rightarrow T_1 &= 36.9K, T_2 = 4.31K \end{aligned} \quad (85)$$

So the maximum inverse temprture is $T = T_1 = 36.9K$

(b) p is a quadratic function of T :

$$\begin{aligned} \frac{\partial p}{\partial T} = 0 &\Rightarrow 5.44 - 0.264T = 0 \Rightarrow T = 20.606K \\ \Rightarrow p_{max} &= 35.05atm \end{aligned} \quad (86)$$

2.30 (10 points) According to our textbook,

$$T_{imax} = \frac{2a}{Rb} = \frac{1.013 \times 10^5 \times 2 \times 1.35 \times 10^{-6}}{8.314 \times 3.1 \times 10^{-5}} K = 1.061 \times 10^3 K \quad (87)$$

2.31 (10 points) For this cycle, the total work the system does to the environment is

$$\begin{aligned} W_{loop} &= W_{AB} + W_{BC} = \int_A^B p dV + \int_B^C p dV \\ &= \int_A^B \frac{RT}{V} dV + p_B(V_C - V_B) = RT_B \ln \frac{V_B}{V_A} + p_B(V_C - V_B) \\ &= RT_B \ln 2 - \frac{RT_B}{2} \end{aligned} \quad (88)$$

while the total heat absorbed can be divided into 2 parts: CA and AB

$$\begin{aligned} Q_{absorb,loop} &= Q_{CA} + Q_{AB} = C_{V,m}(T_A - T_C) + (-W_{AB,ados}) \\ &= C_{V,m}(T_A - T_C) + W_{AB} = \frac{3R}{2}(T_B - \frac{1}{2} \frac{V_B p_B}{R}) + RT_B \ln 2 \\ &= \frac{3R}{4} T_B + RT_B \ln 2 \end{aligned} \quad (89)$$

Thus, the efficiency is:

$$\eta = \frac{W_{loop}}{Q_{absorb,loop}} = \frac{\ln 2 - \frac{1}{2}}{\ln 2 + \frac{3}{4}} = 13.38\% \quad (90)$$

2.32 (10 points)

Proof:

1st, calculate the total work the system does to environment:

$$\begin{aligned} W_{loop} &= W_{12} + W_{23} + W_{34} + W_{41} \\ &= \int_1^2 p dV + \int_2^3 p dV + \int_3^4 p dV + \int_4^1 p dV \\ &= \int_1^2 \frac{p_1 V_1^\gamma}{V^\gamma} dV + p_2(V_3 - V_2) + \int_3^4 \frac{p_4 V_4^\gamma}{V^\gamma} dV + p_1(V_1 - V_4) \\ &= p_1 V_1^\gamma \frac{V_2^{1-\gamma} - V_1^{1-\gamma}}{1-\gamma} + p_2(V_3 - V_2) + p_4 V_4^\gamma \frac{V_4^{1-\gamma} - V_3^{1-\gamma}}{1-\gamma} + p_1(V_1 - V_4) \\ &= \frac{p_2 V_2 - p_1 V_1}{1-\gamma} + \frac{p_4 V_4 - p_3 V_3}{1-\gamma} + (p_3 V_3 - p_2 V_2 + p_1 V_1 - p_4 V_4) \\ &= \frac{\nu R(T_2 - T_1 + T_4 - T_3)}{1-\gamma} + \nu R(T_1 + T_3 - T_2 - T_4) \\ &= \frac{\gamma \nu R(T_2 - T_1 + T_4 - T_3)}{1-\gamma} \\ &= \frac{\gamma \nu R(T_3 - T_2 + T_1 - T_4)}{\gamma - 1} \end{aligned} \quad (91)$$

2nd, calculate the heat absorbed:

$$\begin{aligned} Q_{loop,absorbed} &= Q_{23} = C_p(T_3 - T_2) \\ &= \frac{\gamma \nu R}{\gamma - 1}(T_3 - T_2) \end{aligned} \quad (92)$$

where we use the following relations:

$$\begin{aligned} p_1 V_1^\gamma &= p_2 V_2^\gamma \\ p_3 V_3^\gamma &= p_4 V_4^\gamma \\ \Rightarrow \frac{V_2^\gamma}{V_1^\gamma} &= \frac{p_1}{p_2}, \frac{V_4^\gamma}{V_3^\gamma} = \frac{p_3}{p_4} = \frac{p_2}{p_1} \end{aligned} \quad (93)$$

and

$$\begin{aligned} C_p - C_V &= \nu R \\ \gamma &= \frac{C_p}{C_V} \\ \Rightarrow C_p &= \frac{\gamma \nu R}{\gamma - 1} \end{aligned} \quad (94)$$

and the equation of state.

Then, one gets the efficiency:

$$\eta = \frac{W_{loop}}{Q_{loop,absorbed}} = \frac{T_3 - T_2 + T_1 - T_4}{T_3 - T_2} = 1 - \frac{T_4 - T_1}{T_3 - T_2} \quad (95)$$

2.33 (10 points) One can get :

$$\begin{aligned} p_1 V_1 &= \nu R T_1 = \nu R T_2 = p_2 V_2 \\ p_3 V_3 &= \nu R T_3 = \nu R T_4 = p_4 V_4 \\ V_1 &= V_4, V_2 = V_3 \\ \Rightarrow p_1 p_3 &= p_2 p_4 \end{aligned} \quad (96)$$

2.34 (10 points) For Carnot cycle, the efficiency is:

$$\eta = 1 - \frac{T_{cooler}}{T_{hotter}} = \frac{W}{Q_{absorb}} \quad (97)$$

(a)

According to our textbook, the origin work is

$$W_0 = Q_{absorb} - Q_{release} = \nu R (T_h - T_l) \ln \frac{V_3}{V_4} \quad (98)$$

And later, the work is:

$$\begin{aligned} W' &= \nu R (T'_h - T_l) \ln \frac{V_3}{V_4} = 2W \\ \Rightarrow T'_h &= 2T_h - T_l = 2 \times 373K - 273K = 473K \end{aligned} \quad (99)$$

(b) Thus, the efficiency is:

$$\eta' = 1 - \frac{T_l}{T'_h} = 1 - \frac{273}{473} = 42.3\% \quad (100)$$

2.35 (10 points) The work environment did in the loop is:

$$\begin{aligned} W &= - \int_1^a p dV - \int_2^b p dV = - \int_a^b \frac{\nu R T_a}{V} dV - \int_c^d \frac{\nu R T_d}{V} dV \\ &= \nu R (T_a - T_d) \ln \frac{V_a}{V_b} \end{aligned} \quad (101)$$

And the heat absorbed is

$$Q_{absorb} = W_{cd} = \nu R T_d \ln \frac{V_a}{V_b} \quad (102)$$

Thus, the coefficient is:

$$\epsilon = \frac{Q_{absorb}}{W} = \frac{T_d}{T_a - T_d} = \frac{T_2}{T_1 - T_2} \quad (103)$$

Note: Here we have ignored the heat absorbed during process da and process bc because there is a so-called regenerator during the loop in reality and the heat absorbed and released in the isochoric can be considered as the internal heat exchange.

2.36 (10 points)

2 Ch4

4.1 (1) According to $p = nkT$:

$$n = \frac{p}{kT} = \frac{1.0 \times 101325 Pa}{1.38 \times 10^{-23} J \cdot K^{-1} \times 300 K} = 2.45 \times 10^{25} m^{-3} \quad (104)$$

(2) The average volume of oxygen molecule is $\frac{1}{n}$ and the molecule can be taken as a cube. Thus, the average distance is

$$d = \sqrt[3]{\frac{1}{n}} = \left(\frac{1}{2.45 \times 10^{25}} \right)^{\frac{1}{3}} m = 3.443 \times 10^{-9} m \quad (105)$$

(3)

$$\bar{E}_k = \frac{3kT}{2} = \frac{3 \times 1.38 \times 10^{-23} \times 300}{2} J = 6.21 \times 10^{-21} J = 0.0388 eV \quad (106)$$

4.2 (1) Since $p = nkT$ and p, T are same, so the density of the number of molecule equals.

(2) $m = \rho V = \rho$, so the masses are different.

(3) $\epsilon_k = \frac{3kT}{2}$. Equals.

4.3 (1)

$$n = \frac{p}{kT} = \frac{101325}{1.38 \times 10^{-23} \times 373} m^{-3} = 1.968 \times 10^{19} cm^{-3} \quad (107)$$

(2)

$$\Gamma = \frac{1}{4} \bar{n} \bar{v} = \frac{1}{4} n \sqrt{\frac{8kT}{\pi m}} = \frac{1}{4} \times 1.968 \times 10^{25} \times \sqrt{\frac{8 \times 1.38 \times 10^{-23} \times 373}{\pi \times 18 \times 1.67 \times 10^{-27}}} m^{-2} \cdot s^{-1} = 3.249 \times 10^{27} m^{-2} \cdot s^{-1} \quad (108)$$

(3) For the saturated vapor, there exists a state of equilibrium. So,

$$\Gamma' = \Gamma = 3.249 \times 10^{27} m^{-2} \cdot s^{-1} \quad (109)$$

(4)

$$\bar{\epsilon}_k = \frac{3kT}{2} = \frac{3 \times 1.38 \times 10^{-23} \times 373}{2} J = 7.721 \times 10^{-21} J = 0.048 eV \quad (110)$$

The energy needed for evaporation per molecule is:

$$\begin{aligned} \epsilon &= Lm_0 + p\Delta V = 2250 \times 18 \times 1.67 \times 10^{-27} \times 10^3 J \\ &+ 101325 \times 1670 \times 10^{-6} \times 18 \times 1.67 \times 10^{-24} J = 7.272 \times 10^{-20} J = 0.454 eV. \end{aligned} \quad (111)$$

One can tell the energy needed for evaporation is much larger than thermal motion. And if one heats the system consistently to, say, 1000K, the outermost electrons of the gas molecule can escape, which is called thermionic emission.

4.4

4.5 (1)

$$\bar{v} = \frac{2 \times 1 + 4 \times 2 + 6 \times 3 + 8 \times 4 + 2 \times 5}{2 + 4 + 6 + 8 + 2} m \cdot s^{-1} = 3.182 m \cdot s^{-1} \quad (112)$$

(2)

$$\sqrt{\bar{v}^2} = \sqrt{\frac{2 \times 1^2 + 4 \times 2^2 + 6 \times 3^2 + 8 \times 4^2 + 2 \times 5^2}{2 + 4 + 6 + 8 + 2}} m \cdot s^{-1} = 3.371 m \cdot s^{-1} \quad (113)$$

(3)

$$v_p = 4m \cdot s^{-1} \quad (114)$$

4.6

$$N_1 = \Delta v f(v_1) \quad (115)$$

$$N_2 = \Delta v f(v_2) \quad (116)$$

Thus,

$$\begin{aligned} \frac{N_1}{N_2} &= \frac{f(v_1)}{f(v_2)} = \exp\left(-\frac{m(v_1^2 - v_2^2)}{2kT}\right) \frac{v_1^2}{v_2^2} = 4 \exp\left(-\frac{2 \times 1.673 \times 10^{-27} (3000^2 - 1500^2) J}{2 \times 1.38 \times 10^{-23} \times 300 J}\right) \\ &= 0.261 \end{aligned} \quad (117)$$

4.7 For the adiabatic process, $dQ = 0$ and $U = u(T)V = 3pV$

$$3d(pV) + pdV = 0$$

$$4pdV + 3Vdp = 0$$

$$\ln(V^4 p^3) = C_1$$

$$pV^{\frac{4}{3}} = C \quad (118)$$

4.8 (1)

$$\begin{aligned} f(v)dv &= 4\pi\left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}}v^2e^{-\frac{mv^2}{2kT}}dv = f(x)dx \\ \Rightarrow f(x) &= \frac{dv}{dx}f(v) = 4\pi v_p\left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}}x^2v_p^2e^{-\frac{mx^2v_p^2}{2kT}} \end{aligned} \quad (119)$$

given that $v_p^2 = \frac{2kT}{m}$, one has:

$$f(x) = 4\pi\left(\frac{2kT}{m}\right)^{\frac{3}{2}}\left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}}x^2e^{-\frac{mx^2}{2kT} \cdot \frac{2kT}{m}} = \frac{4}{\sqrt{\pi}}x^2e^{-x^2} \quad (120)$$

(2)

$$\begin{aligned} f(v)dv &= 4\pi\left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}}v^2e^{-\frac{mv^2}{2kT}}dv = f(\epsilon_k)d\epsilon_k \\ \Rightarrow f(\epsilon_k) &= 4\pi\left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}}v^2e^{-\frac{mv^2}{2kT}}\frac{dv}{d\epsilon_k} = 4\pi\left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}}v^2e^{-\frac{mv^2}{2kT}}\sqrt{\frac{1}{2m\epsilon_k}} \\ &= 4\pi\left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}}\frac{2\epsilon_k}{m}\sqrt{\frac{1}{2m\epsilon_k}}e^{-\frac{\epsilon_k}{kT}} = \frac{2}{\sqrt{\pi}(kT)^{\frac{3}{2}}}\sqrt{\epsilon_k}e^{-\frac{\epsilon_k}{kT}} \end{aligned} \quad (121)$$

Hint: one should think about the conservation of probability: $f(x)dx = \pm g(y)dy$ (for one-dimension).

4.9 According to 4.8, $f(x) = \frac{4}{\sqrt{\pi}}x^2e^{-x^2}$,

$$\begin{aligned} \frac{n}{N} &= \int_1^2 f(x)dx = \int_1^2 \frac{4}{\sqrt{\pi}}x^2e^{-x^2}dx = 0.233253 \times \frac{4}{\sqrt{\pi}} = 0.5264 \\ \Rightarrow n &= 0.5264N \end{aligned} \quad (122)$$

Using Mathematica, one can compute the integral.

4.10 Using $p = p_0e^{-\frac{mgz}{kT}}$:

$$z = \frac{kT}{mg} \ln \frac{p_0}{p} = \frac{kN_A \times 278.15K}{28 \times 10^{-3} \times 9.8N} \ln \frac{760}{590} = 2133.84m \quad (123)$$

4.11 According to $n = n_0e^{-\frac{mgz}{kT}}$, when n comes to $\frac{1}{2}n_0$:

$$z = \frac{kT}{mg} \ln 2 = \frac{RT}{\mu g} \ln 2 = \frac{8.314 \times 300}{29 \times 10^{-3} \times 9.8} \ln 2 = 6.083 \times 10^3m \quad (124)$$

4.12 The distribution of classical particles in a given potential obeys Boltzman distribution, whose probability density (separating its kinetic terms and considering only 1-dimension) is:

$$f(z) = \frac{n_0}{N}e^{-\frac{mgz}{kT}} \quad (125)$$

where $\frac{n_0}{N} = \frac{mg}{kT} \frac{1}{1 - e^{-\frac{mgL}{kT}}}$.

Thus, the average potential is:

$$\begin{aligned}\bar{\epsilon}_p &= \int_0^L f(z)mgzdz = \frac{n_0mg}{N} \int_0^L ze^{-\frac{mgz}{kT}} dz = \frac{n_0(kT)^2}{Nmg} \int_0^{\frac{mgL}{kT}} xe^{-x} dx \\ &= \frac{kT}{1 - e^{-\frac{mgL}{kT}}} \left(1 - \left(1 + \frac{mgL}{kT}\right)e^{-\frac{mgL}{kT}}\right) \\ &= kT - \frac{mgL}{e^{\frac{mgL}{kT}} - 1}\end{aligned}\quad (126)$$

As for the average kinetic energy, it is the same as what we are talking about as usual:

$$\bar{\epsilon}_k = \frac{3kT}{2}\quad (127)$$

4.13 (1)

$$\bar{v} = \sqrt{\frac{8kT}{\pi m}} = \sqrt{\frac{8RT}{\pi \mu}} = \sqrt{\frac{8 \times 8.314 \times 373}{\pi \times 200.59 \times 10^{-3}}} m \cdot s^{-1} = 198.41 m \cdot s^{-1}\quad (128)$$

(2) Using $p = nkT$,

$$\begin{aligned}\Delta N &= \frac{1}{4}n\bar{v}\Delta S\Delta t = \frac{1}{4}\frac{p}{kT}\bar{v}\pi\left(\frac{d}{2}\right)^2\Delta t \\ &= \frac{1}{4} \times \frac{37.33}{1.38 \times 10^{-23} \times 373} \times 198.41 \times \pi \times (0.1 \times 10^{-3})^2 \times 3600 h^{-1} \\ &= 4.068 \times 10^{19} h^{-1}\end{aligned}$$

and

$$\Delta M = \frac{\Delta N}{N_A}\mu = 0.01356 g \cdot h^{-1}\quad (129)$$

4.14 Given that the temperature remains constant, the average velocity remains unchanged, too.

$$\begin{aligned}dN &= -\Gamma A dt = -\frac{1}{4}\bar{v}\frac{N}{V}A dt \\ \Rightarrow \ln N &= -\frac{\bar{v}At}{4V} + C \\ \Rightarrow N &= N_0 e^{-\frac{\bar{v}At}{4V}}\end{aligned}\quad (130)$$

Hint: It is very similar to the last problem in our first quiz.

4.15 Proof:

$$\begin{aligned}\bar{\epsilon}_{k,x} &= \frac{1}{2}m\bar{v}_x^2 = \int dv_x dv_y dv_z f(\mathbf{v})\frac{1}{2}mv_x^2 \\ &= \frac{1}{2}m\left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} \int dv_x dv_y dv_z e^{-\frac{m(v_x^2+v_y^2+v_z^2)}{2kT}} v_x^2\end{aligned}\quad (131)$$

Using $\int dv_y e^{-\frac{mv_y^2}{2kT}} = \sqrt{\frac{2kT}{m}} \int dx e^{-x^2} = \sqrt{\frac{2kT}{m}} \sqrt{\pi}$ and

$$\int dv_x e^{-\frac{mv_x^2}{2kT}} v_x^2 = \left(\sqrt{\frac{2kT}{m}}\right)^3 \times 2 \int_0^\infty x^2 e^{-x^2} dx = 2\left(\sqrt{\frac{2kT}{m}}\right)^3 \frac{1}{2} \int_0^\infty dt \sqrt{t} e^{-t} = \left(\sqrt{\frac{2kT}{m}}\right)^3 \Gamma\left(\frac{3}{2}\right),$$

one has:

$$\bar{\epsilon}_{k,x} = \frac{m}{2} \left(\frac{m}{2\pi kT}\right)^{\frac{1}{2}} \left(\sqrt{\frac{2kT}{m}}\right)^3 \Gamma\left(\frac{3}{2}\right) = \frac{1}{2\sqrt{\pi}} 2kT \cdot \frac{1}{2} \sqrt{\pi} = \frac{kT}{2}\quad (132)$$

4.16 For hydrogen gas in 300K, the heat capacity 1 mol is about $\frac{5R}{2}$

$$U_{H_2,1mol} = \frac{5RT}{2} = \frac{5 \times 8.314 \times 300}{2} J = 6.235 \times 10^3 J \quad (133)$$

which is same for 1 mol nitrogen gas.

As for 1g hydrogen gas and nitrogen gas:

$$\begin{aligned} U_{H_2,1g} &= \frac{5mRT}{2\mu_1} = 3.118 \times 10^3 J \\ U_{N_2,1g} &= \frac{5mRT}{2\mu_2} = 222.70 J \end{aligned} \quad (134)$$

4.17 The gas molecules run from one box to the other : $Q = Q_1 + Q_2$, but there only one kind of gas in the two box, so $Q = |Q_1 - Q_2|$, because you cannot tell the ascription of the gas from large scale, but you do find the number of molecules decreases in one box while increasing in the other.

$$\begin{aligned} Q_1 &= \frac{\Gamma A}{N_A} \mu = \frac{1}{4} \frac{\bar{v}_1 n_1 A}{N_A} \mu = \frac{1}{4} \frac{p_1 A \mu}{k T N_A} \sqrt{\frac{8kT}{\pi m}} = \frac{p_1 A \mu}{4RT} \sqrt{\frac{8RT}{\pi \mu}} \\ Q_2 &= \frac{p_2 A \mu}{4RT} \sqrt{\frac{8RT}{\pi \mu}} \\ \Rightarrow Q &= Q_1 + Q_2 = \frac{|p_1 - p_2| A \mu}{4RT} \sqrt{\frac{8RT}{\pi \mu}} \end{aligned} \quad (135)$$

4.18 (1)4-atom molecule: there always 3 degrees of freedom for the translation of center of mass, 3 dof for the rotation of com, and $(3 \times 4 - 6) = 6$ dof for the relative vibration between atoms.

Or equivalently, r=t=3, s=6.

(2)

$$c_{v,m} = \frac{1}{2}(r + t + 2s)R = 9R \quad (136)$$

4.19 (1)t=3

$$\begin{aligned} \bar{\epsilon}_k &= \frac{3kT}{2} = 6.21 \times 10^{-21} J \\ \bar{v} &= \sqrt{\frac{8kT}{\pi m_{H_2}}} = \frac{1}{\sqrt{2}} \times 2.52 \times 10^3 m \cdot s^{-1} = 1.782 \times 10^3 m \cdot s^{-1} \end{aligned} \quad (137)$$

(2)r=2

$$\begin{aligned} \bar{\epsilon}_r &= \frac{2kT}{2} = kT \\ \bar{\epsilon}_r &= \frac{1}{2} \times 2\mu\omega^2 r^2 \end{aligned} \quad (138)$$

Using $\mu = \frac{1}{2}m_{H_2} = m_H$ and $r = \frac{d}{2}$, one gets:

$$\omega = \sqrt{\frac{kT \times 2}{m_H d^2}} = 2.233 \times 10^{13} Hz \quad (139)$$

(3) $t=3,r=2,s=1$: but in 300K the vibration freedom has been frozen, so,

$$\begin{aligned} c_{V,m} &= \frac{1}{2}(t+r)R = \frac{5R}{2} \\ \gamma &= \frac{c_{p,m}}{c_{V,m}} = 1 + \frac{R}{c_{V,m}} = \frac{7}{5} \end{aligned} \quad (140)$$

3 Ch6

6.1 (1) Given that this captivity has water moisture and other ordinary gas above liquid water, one has:

$$p_1 = p_{gas} + p_{mois} = 3atm \quad (141)$$

And $V_1 \rightarrow 2V_1$ with temperature constant, $p_{gas} \rightarrow \frac{1}{2}p_{gas}$, while p_{mois} remains constant. So

$$p_2 = p_{mois} + \frac{1}{2}p_{gas} = 2atm \quad (142)$$

Solve these two equations:

$$\begin{cases} p_{gas} = 2atm \\ p_{mois} = 1atm \end{cases} \quad (143)$$

Since the moisture is water moisture, that the saturated vapor pressure equals to 1atm means the temperature equals to **373K**.

(2)The water has been fully evaporated, so the following process is just like the ideal gas: $V \rightarrow 2V$ and $p \rightarrow \frac{p}{2}$, which means $p_3 = \frac{1}{2}p_2 = 1atm$

6.2

$$\Delta u = Lm - p\Delta V = 8.63 \times 10^5 \times 4 \times 10^{-3} J - 1.013 \times 10^5 \times 0.607 \times 4 \times 10^{-3} J = 3206.04 J \quad (144)$$

Hint: recall problem2.17

6.3 (1)For this kind of gas, when it comes to liquefaction with temperature constant, pv remains constant:

$$v_c = \frac{m}{\rho_c} = \frac{0.027}{4} m^3 = 6.75 \times 10^{-3} m^3 \quad (145)$$

(2)when the liquefaction ends,

$$\begin{cases} \rho_g v_f + \rho_l v_l = m \\ v_g = 0 \end{cases} \Rightarrow v_l = \frac{m}{\rho_l} = \frac{0.027}{4} m^3 = 6.75 \times 10^{-3} m^3 \quad (146)$$

(3)

$$\begin{cases} \rho_g v_f + \rho_l v_l = m \\ v_g + v_l = v_{total} \end{cases} \Rightarrow \begin{cases} v_g = 9.87 \times 10^{-4} m^3 \\ v_l = 1.28 \times 10^{-5} m^3 \end{cases} \quad (147)$$

6.4

$$Q = \Delta u + p\Delta v = Lm \quad (148)$$

let us assume the mass of gas is m , then the proportion is:

$$\eta = \frac{pv}{Lm} = \frac{1.01325 \times 10^5 \times 1.65m}{5.39 \times 10^5 \times 4.184m} = 0.0741 = 7.41\% \quad (149)$$

6.5 According to Clapeyron equation,

$$\begin{aligned} \frac{dp}{dT} &= \frac{L}{T(v_l - v_s)} \\ \Rightarrow \Delta p &= \frac{L\Delta T}{T(v_g - v_s)} = \frac{3.34 \times 10^5 \times (-1)}{273 \times (10^{-3} - 1.0905 \times 10^{-3})} Pa = 1.352 \times 10^7 Pa \simeq 133.4 atm \end{aligned} \quad (150)$$

So the outside pressure is 134.4atm

6.6 Similar to the last problem,

$$\frac{dp}{dT} = \frac{L}{T(v_l - v_s)} = \frac{24.5 \times 10^3}{600 \times \left(\frac{1}{10.65 \times 10^3} - \frac{1}{11.01 \times 10^3}\right)} Pa/K = 1.33 \times 10^7 Pa/K \quad (151)$$

So $\Delta T = \Delta p \frac{dT}{dp} = \frac{1.01 \times 10^7 - 1.013 \times 10^5}{1.33 \times 10^7} K = 0.759K$, and thus the new melting point is $T_0 + \Delta T = 600.759K$

6.7 Using $\frac{dp}{dT} = \frac{L}{T(v_g - v_s)} \simeq \frac{dp}{dT} = \frac{L}{Tv_g}$, (here, we have ignored the volume of solid matter) one gets:

$$v_g dp = \frac{LdT}{T}$$

take the gas as ideal gas:

$$\begin{aligned} \frac{RTdp}{p} &= \frac{LdT}{T} \\ \Rightarrow \ln p &= -\frac{L}{RT} + C \\ \text{or: } \lg p &= -\frac{L}{2.30RT} + D \end{aligned} \quad (152)$$

So $\frac{L}{2.30R} = 6800 \Rightarrow L = 2.30 \times 6800R = 1.300 \times 10^5 J/mol$.

6.8

$$\Delta T = \frac{dT}{dp} \Delta p \simeq \frac{Tv_g}{L} \Delta p = \frac{373.15 \times 1.671}{2.26 \times 10^6} \times 10 \times 133.3K = 0.368K \quad (153)$$

So the new boiling point is about $T + \Delta T = 373.15K + 0.368K = 373.52K$

6.9

$$\begin{aligned} \frac{dp}{dT} &= \frac{L}{T(v_g - v_l)} \simeq \frac{L}{Tv_g} \\ \Rightarrow L &= Tv_g \frac{dp}{dT} \simeq Tv_g \frac{\Delta p}{\Delta T} = 373 \times 1.671 \times \frac{3 \times 133.3}{0.11} J/kg = 2.266 \times 10^6 J/kg \end{aligned} \quad (154)$$

6.10 We should consider the temperature-constant model for the gradient of pressure and using Clapeyron equation to describe the relation between outside temperature and pressure. Then take the gas as ideal gas.

$$\begin{aligned}
& \begin{cases} \frac{dp}{dT} = \frac{L}{Tv_g} = \frac{pL}{RT^2} \\ \frac{dp}{dz} = -\rho g = -nmg = -\frac{pmg}{kT_0} \end{cases} \Rightarrow \frac{dp}{dz} = \frac{dp}{dT} \frac{dT}{dz} = -\frac{pmg}{kT_0} \\
& \Rightarrow \frac{dT}{dz} = -\frac{pmg}{kT_0} \frac{RT^2}{pL} = -\frac{mgN_A T^2}{T_0 L} \\
& \Rightarrow -\frac{1}{T} = -\frac{mgN_A}{LT_0}(z - z_0) + C \\
& \Rightarrow \frac{1}{T} = \frac{1}{T_{sealevel}} + \frac{mgN_A}{LT_0}z = \frac{1}{373.15K} + \frac{mgN_A}{LT_0}z \quad (155)
\end{aligned}$$

6.11 For 1-order phase transition, the chemical potential, pressure and temperature remain constant:

$$\begin{aligned}
& d\mu = du - Tds + pdv = 0 \\
& \Rightarrow du = Tds - pdv \\
& \Rightarrow \Delta u = T\Delta s - p\Delta v = L - p \frac{LdT}{Tdp} = L(1 - \frac{\ln T}{\ln p}) \quad (156)
\end{aligned}$$

where we used Clapeyron equation: $\Delta v = \frac{LdT}{Tdp}$.

6.12 Refer to our textbook,

$$\begin{cases} T_c = \frac{8a}{27bR} \\ p_c = \frac{a}{27b^2} \end{cases} \Rightarrow \begin{cases} T_c = \frac{8 \times 1.35 \times 10^{-6} \times 1.013 \times 10^5}{27 \times 3.1 \times 10^{-5} \times 8.314} K = 157.22K \\ p_c = \frac{1.35 \times 10^{-6} \times 1.013 \times 10^5}{27 \times (3.1 \times 10^{-5})^2} Pa = 5.27 \times 10^6 Pa = 52.02 atm \end{cases} \quad (157)$$

Note that one should set 1mol.

4 Reference

- [1] 张玉民, 等. 热学 [M]. 北京: 科学出版社, 2006.
- [2] 朱晓东, 等. 热学 [M]. 合肥: 中国科学技术大学出版社, 2014.5
- [3] 郑久仁, 等. 热学热力学统计物理 [M]. 合肥: 中国科学技术大学出版社, 2005
- [4] 汪志诚, 等. 热力学, 统计物理 [M]. 北京: 高等教育出版社, 2012